UCRL- 93035
PREPRINT

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This paper was prepared for submittal to
Physical Review C

July 15, 1985

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PACS number: 21.60.-n, 21.60.Ev, 21.60.Fw 25.55.Ek, 23.20.Js

Abstract

The Interacting Boson Model is able to reproduce the elusive two particle transfer data in the Actinides (uranium and thorium). The same calculation also reproduces well the energy systematics and the B(E2) data.

Key words: Interacting Boson Model, Actinides, Spectra, B(E2)'s, Two Neutron Transfer Reactions

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Two neutron transfer reactions in the actinides have not been well understood since the first experimental data[1] became available. These data showed that the cross section for the reactions A+2(p,t)A populating the 0^+_2 state (i.e. the β band head) was an appreciable fraction, typically 10-15% of the cross section for population of the 0^+_1 ground state by the same reaction. By contrast, predictions based on the geometrical model give a very low estimate for the ratio of these two cross sections ($\approx 10^{-3}$). On the other hand, it was found that the experimental cross-section ratio for populating these same bands by the A(t,p)A+2 reaction was generally quite small ($\approx 10^{-3}$), although ratios comparable to those found for the (p,t) reaction were observed for a few product nuclei, e.g. ^{248}Cm[2]. This is well summarized in a recent publication by Janecke et. al[3].

Janecke et. al.[3] have also estimated the ratio of the two neutron transfer (TNT) cross sections to the band head of the β-band and the ground state in the actinides using the Interacting Boson Model (IBM)[4]. Their ratios are similar to those based on the geometrical model, i.e. two orders of magnitude lower than experimental values for (p,t) reactions. Their result have suggested that TNT reactions in the actinides cannot be described by the IBM. We show below that this is not the case. It is shown in this paper that the TNT cross sections depend critically on the choice of the IBM Hamiltonian. We have chosen a Hamiltonian different in form from that adopted by Janecke et al. Using this Hamiltonian we have reproduced the energy spectra and B(E2) values equally well (or better) and have also been able to reproduce the TNT cross ratios for (p,t) reactions.

In the Interacting Boson Model, the even parity collective excitations can be described by a Hamiltonian depending on six parameters[4]. In principle, these parameters can be determined for any nucleus by fitting model predictions to experimental data using least squares procedures. Typically, the fit is more sensitive to some parameters and less sensitive to others. Because of the complexity of the problem, a restricted class of Hamiltonians is generally studied and the parameters in this class are optimized. Because of the insensitivity of the spectra to some parameter variations, it may happen that quite different Hamiltonians give rise to spectra which fit experimental energy levels with roughly comparable residuals. In such cases, one must distinguish among models by probing the eigenfunctions. All such probes involve transitions. The most sensitive and most used probes are the B(E2) values for transitionss that depopulate the β and γ bands. Another useful probe is the TNT reactions.

The Hamiltonian used by Janecke et. al.[3] to fit the spectra in the actinide region belongs to a restricted class of the form

$$\mathbf{H}_{\epsilon} = \epsilon_d \mathbf{n}_d + \kappa \mathbf{Q} \cdot \mathbf{Q} \tag{1}$$

where $\mathbf{n}_d$ is the d-boson number operator and

$$\mathbf{Q} = (\mathbf{d}^{\dagger}\mathbf{s} + \mathbf{s}^{\dagger}\mathbf{d})^2 + \chi/\sqrt{5}\,(\mathbf{d}^{\dagger}\mathbf{d})^2 \tag{2}$$

We have recently studied[5] the properties of another restricted class of Hamiltonian of the form

$$\mathbf{H}_{\chi} = -\kappa \mathbf{Q}.\mathbf{Q} + \kappa' \mathbf{L}.\mathbf{L} \tag{3}$$

$$\mathbf{L} = \sqrt{10}\,(\mathbf{d}^{\dagger}\mathbf{d})^1. \tag{4}$$

The operators $\mathbf{s}^{\dagger},\mathbf{d}^{\dagger}/\mathbf{s},\mathbf{d}$ in eqs.(1) through (4) are the creation/annihilation operators for the l=0(s) and 2(d) bosons. When $\chi = -\sqrt{35}/2(0)$, the operator $\mathbf{Q}$ of eq. (3) is a generator of the SU(3)(O(6)) algebra. Both Hamiltonians can reproduce spectra in reasonable agreement with the experimental spectra (with exceptions noted below), with the fits using eq.(2) somewhat better than those obtained from eq.(1). However, <u>it must be noted that when the experimental energy levels exhibit a nearly SU(3) behavior.</u>

eq.(1) cannot adequately fit the data.

We have tested these two Hamiltonians against the thorium and uranium isotopes for four reasons:

i) these isotopes have well-defined β- and γ- bands;

ii) there are no 0^+ intruder state below 1.5 MeV;

iii) there are good measurements of two neutron transfer reactions.

iv) the β- and γ- bands are rotational bands with moments of inertia slightly greater than that of the ground band.

The spectra of these nuclei have also been studied in Ref. 6 by the use of a general IBM Hamiltonian with six parameters. In this paper, we shall compare the results obtained by the use of the Hamiltonians defined by eqs. (1) and (3). In Fig.1, we compare the best fits obtained from the Hamiltonians of eqs.(3) to the experimental data for ^{232}U, ^{234}U and ^{236}U (Fig 1a) as well as for ^{230}Th and ^{232}Th (Fig. 1b).

The eigenvectors associated with experimental states are significantly different for these two Hamiltonians. This is easily seen in Table 1. In this Table, we display the overlap integrals between the three lowest 0^+ states computed with $\mathbf{H}_\epsilon$ and $\mathbf{H}_\chi$ We also display the inner products for the four lowest 2^+ states. The parameters for the Hamiltonians $\mathbf{H}_\epsilon$ and $\mathbf{H}_\chi$ were adjusted to provide a best fit to the experimental energy levels for ^{232}U. These results show clearly that the eigenfunctions for corresponding physical states computed using the two Hamiltonians are very dissimilar. In fact, the β-band head $(0^+{}_2)$ computed using $\mathbf{H}_\epsilon$ looks more like the ground state $(0^+{}_1)$ than the $0^+{}_2$ state computed using $\mathbf{H}_\chi$,and conversely. Despite this, the energy eigenvalues for corresponding states are comparable, except for the near perfect SU(3) nuclei ^{232}Th and ^{236}U. The 2^+ states also display the same characteristics as the 0^+ states.

The B(E2)s consitute another probe of the wavefunctions. In lowest order, the transition operator can be chosen as

$$\mathbf{T}(E2) = e\ \{ (\mathbf{d}^{\dagger}\mathbf{s} + \mathbf{s}^{\dagger}\mathbf{d})^2 + \chi_{E2}/\sqrt{5}(\mathbf{d}^{\dagger}\mathbf{d})^2 \} = e\mathbf{Q}_{E2} \qquad (5)$$

Due to the necessary tensorial property, the operator $\mathbf{T}(E2)$ in eq.(5) has the same form as in eq.(2). However, the parameter $\chi_{E2} \neq \chi$ stems from the different physical origin of the operators, one electromagnetic, the other nuclear[7]. We have computed the B(E2) transitions using the wavefunctions of $\mathbf{H}_{\epsilon}$ and $\mathbf{H}_{\chi}$; the results are presented in Table 2 for ^{234}U. The effective charge is adjusted so as to reproduce the $2^+_g \to 0^+_g$ transitions. The expectation value of the B(E2) operator of eq. (5) in the wave functions of the two hamiltonians is a function of χ_{E2}; the $2^+_{\gamma} \to 0^+_g$ transition is consistent with the choice $\chi_{E2} = 0$ for both Hamiltonians. On the other hand, the $2^+_{\beta} \to 0^+_g$ transition is not as well reproduced (see table2).

These wavefunctions can also be probed by TNT reactions. The simplest TNT operator describing transitions from the 0^+ ground state to 0^+ states is proportional to $\mathbf{s}^{\dagger}$ ($\mathbf{s}$) for TNT stripping (pickup) reactions (or $\mathbf{s}$ ($\mathbf{s}^{\dagger}$) above half-shell). These operators must be modified by correction factors which take account of the indistinguishability of neutron and proton bosons in the IBM. We have computed the ratio for transfer into the 0^+_2 states and the 0^+_1 state as follows. The experimental energy spectra are used to determine the values of the parameters in the Hamiltonians using a least squares method. The matrix elements of the two-neutron transfer operator $\mathbf{s}$ ($\mathbf{s}^{\dagger}$) between the 0^+_1 ground state of A and the 0^+_2 and 0^+_1 states of A-2 (A+2) are computed, and the ratio of their absolute squares is taken. This was done for both Hamiltonians $\mathbf{H}_{\epsilon}$ and $\mathbf{H}_{\chi}$. For $\mathbf{H}_{\epsilon}$, these ratios are 10^{-3}, in agreement with the results of Janecke et. al.[3] but in disagreement with the data. For $\mathbf{H}_\chi$, the ratios are consistent with experimental values. These values are presented in Table 3. Note that for an arbitrary χ, the Hamiltonian $\mathbf{H}_\chi$ does not follow a dynamical symmetry. Consequently, the TNT cross sections depend, via the wavefunctions, the choice of χ.

The Hamiltonian $\mathbf{H}_\chi$ has been shown to be applicable in the rare earth region, where the β-band generally occurs above the γ-band (exception: ^{172}Hf)[5]. In particular, it is able to describe series of isotopes in which band inversion occurs (e.g. ^{168}Hf - ^{172}Hf). Band inversion occurs as the parameter χ crosses through the SU(3) limiting value $-\sqrt{35}/2$. In the present paper, we show the applicability of this Hamiltonian in the actinide region, where the β-band generally occurs below the γ-band (exceptions: ^{224}Ra and heavier Cm isotopes). We have now shown that this Hamiltonian is not only useful at describing spectra throughout this region, but seems to provide reasonable wavefunctions and may resolve a long-standing question about the "anomalous" strength of the two-neutron transfer cross sections into the β-band head.

We thank Ron Brown, John Erskine and J. L. Wood for their communications about the TNTR data. We thank also R. F. Casten, J. Janecke and B. H. Wildenthal for useful discussions. All numerical computations were done with the code PHINT and FTNT by O. Scholten who has kindly given us permission to use them. This work is supported by the National Science Foundation (PHY-844-1891), Department of Energy (W-7405-ENG-48) and the Chinese Science Foundation.

*This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.

Table Captions

Table 1. The overlap integrals of the three lowest 0+ states and the four lowest 2^+ states for ^{232}U computed using the Hamiltonians $\mathbf{H}_\chi$ and $\mathbf{H}_\epsilon$ are shown.

Table 2. The B(E2) transitions for ^{234}U. The calculated values are the expectation values of the $\mathbf{T}(E2)$ operator, with $\chi_{E2} = 0$, with respect to the wavefunctions of $\mathbf{H}_\epsilon$ and $\mathbf{H}_\chi$.

Table 3. Experimental and theoretical cross section ratios for (t,p) and (p,t) reactions. (a) The (p,t) L=0 cross section ratios taken from ref.1. (b) The (t,p) L=0 cross section ratios taken from ref. 7. (c) The nucleus 232Th is a good SU(3) nucleus and is difficult to fit with $\mathbf{H}_\epsilon$. ^{236}U is not as good an SU(3) nucleus and has been fitted as well as possible by $\mathbf{H}_\epsilon$.

Figure caption

Figure 1 : Comparison between the experimental spectra and the best fit using the Hamiltonian (3). a) Uranium isotopes. b) Thorium isotopes. The PHINT parameter values (N,CHQ,ELL,QQ) are: ^{232}U (12,-5.206,0.00925, -0.11433); ^{234}U (13,-3.740,0.0070,-0.019); ^{236}U (14,-2.933,0.00064, -0.0228); ^{239}Th (11,-4001,0.0100,-0.0150); ^{232}Th (12,-3.065,0.0084, -0.0209)

Table 1

H_ϵ \ H_χ	0^+_1	0^+_2	0^+_3
0^+_1	0.525	0.590	-0.453
0^+_2	0.536	0.181	0.207
0^+_3	0.422	-0.119	0.338

H_ϵ \ H_χ	2^+_1	2^+_2	2^+_3	2^+_4
2^+_1	0.533	0.590	0.007	0.460
2^+_2	0.549	0.164	-0.031	-0.245
2^+_3	0.060	0.009	0 404	-0.031
2^+_4	-0.234	0.115	0.481	0.148

Table 2

B(E2)	Exp(spu)	H_{ϵ}	H_{χ}
Effective charge		1.30	1.59
$2^+_g \rightarrow 0^+_g$	51.2 ± 0.5	51.2	51.2
$2^+_\beta \rightarrow 0^+_g$	2.3 ± 0.3	0.033	0.69
$2^+_\gamma \rightarrow 0^+_g$	2.9 ± 0.3	3.46	2.54

Table 3

	^{232}U		^{234}U		^{236}U
		(p,t) $\longleftarrow$		(p,t) $\longleftarrow$	
Exp		0.13 ± 0.01 [a]		0.13 ± 0.01 [a]	
H_ϵ		0.001		0.0006	
H_χ		0.12		0.06	
		(t,p) $\longrightarrow$		(t,p) $\longrightarrow$	
Exp				< 0.018 [b]	
H_ϵ		0.15		0.17	
H_χ		0.03		0.06	

	^{230}Th		^{232}Th
		(p,t) $\longleftarrow$	
Exp		0.18 ± 0.02 [a]	
H_ϵ		c)	
H_χ		0.135	
		(t,p) $\longrightarrow$	
Exp		0.028 [b]	
H_ϵ		c)	
H_χ		0.135	

a) The (p,t) L=0 cross section ratios taken from ref. 1.

b) The (t,p) L=0 cross section ratios taken from ref. 7.

c) The nucleus ^{232}Th is a good SU(3) nucleus and is difficult to fit with $\mathbf{H}_{\epsilon}$. On the other hand, ^{236}U is not as good an SU(3) nucleus and has been fitted as well as possible by $\mathbf{H}_{\epsilon}$.

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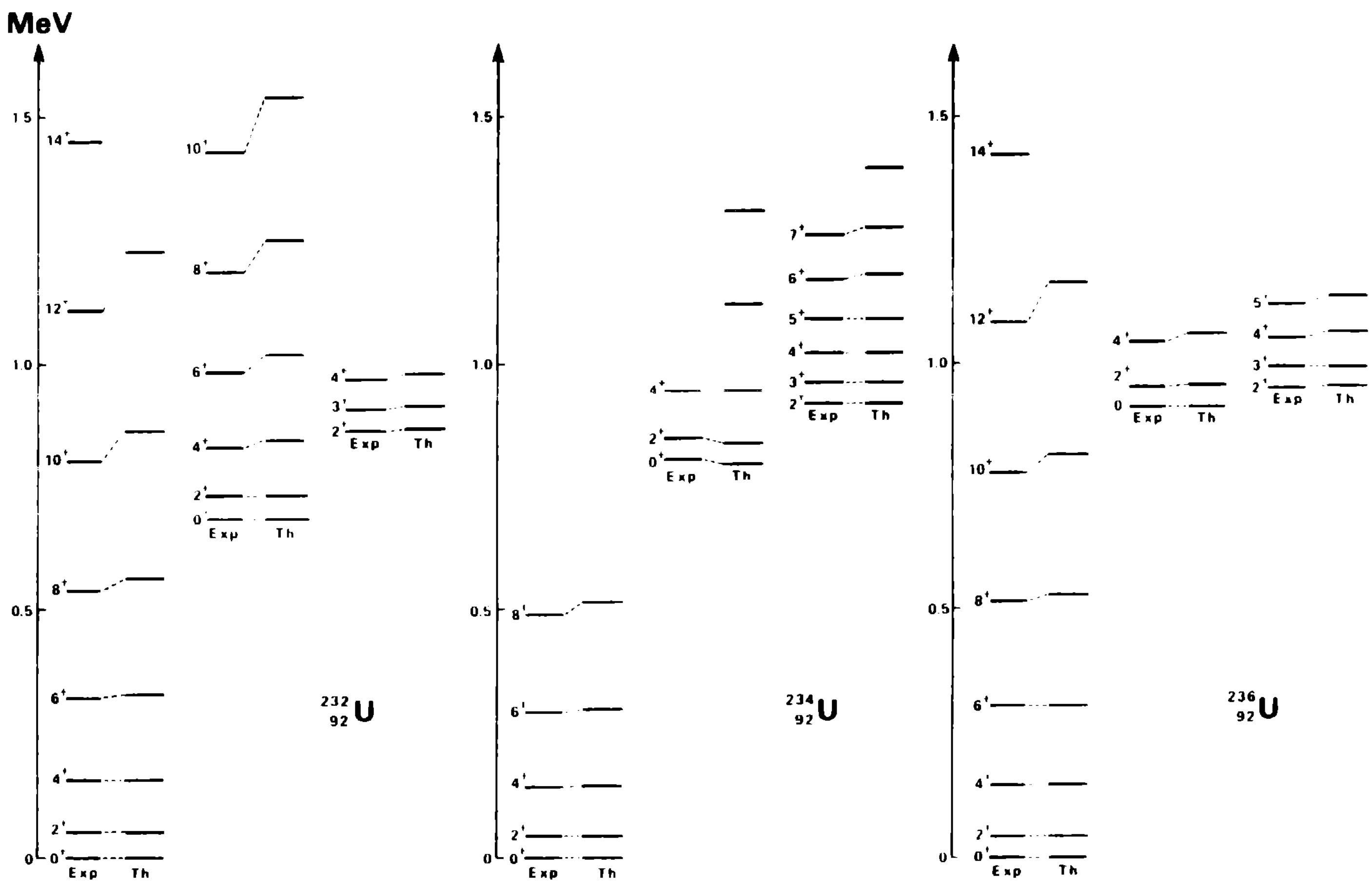
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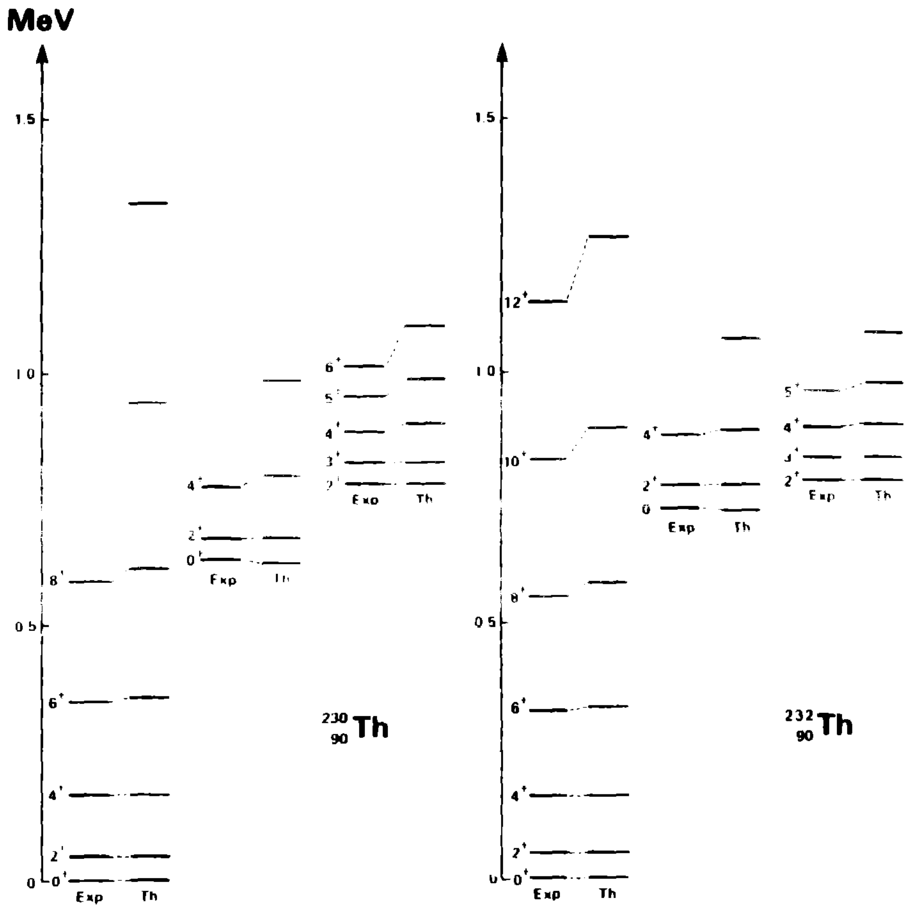

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